Dynamic Algebra in EpsilonWriter

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MC^2 European project

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Dynamic Algebra in EpsilonWriter

Dynamic Algebra: Calculations by direct manipulation

Initial question:

When can we move a sub-expression in a formula, getting an equivalent formula?

Basic examples:

- $\mathbf{2} + 3x + y \rightarrow 3x + y + \mathbf{2}$ additive sub-expression unchanged
- $\mathbf{2}\frac{x}{3} \rightarrow \frac{\mathbf{2}x}{3}$ multiplicative sub-expression unchanged
- $\mathbf{2} + 3x = 5 \rightarrow 3x = 5 \mathbf{-} \mathbf{2}$ additive sub-expression changed to opposite
- $\mathbf{2}x = 5 \rightarrow x = \frac{5}{\mathbf{2}}$ multiplier changed to divisor
- $\mathbf{-2}x < 5 \rightarrow x > \frac{5}{\mathbf{-2}}$ multiplier changed to divisor, inversion of the inequality

First Level of Dynamic Algebra
The theory of movements in formulas (TMF)

- TFM considers 3 statuses: *adder*, *multiplier* and *divisor* (no status is attached to the subtraction, $a - b$ is seen as the sum of a and $-b$)
- TMF considers the movements of a sub-expression *u* through operators (+ – × / and relations)

The question is:

Is it possible to move u in a formula, u being unchanged (with same status or not) or changed to $-u$, the rest of the formula being unchanged except possibly the orientation of an inequality?

The theory of movements in formulas (TMF)

For the fundamental operators + – × / and relations, within the considered status and classes of movements, TMF identifies 74 cases and 56 of them produce correct transformations, which corresponds to **76% of correct transformations**

See http://www.epsilonwriter.com/TMF/TMF-initiale.pdf

This percentage leads us to consider that the notion of movement of sub-expressions in formulas is a meaningful concept of algebraic manipulations.

Pedagogical studies

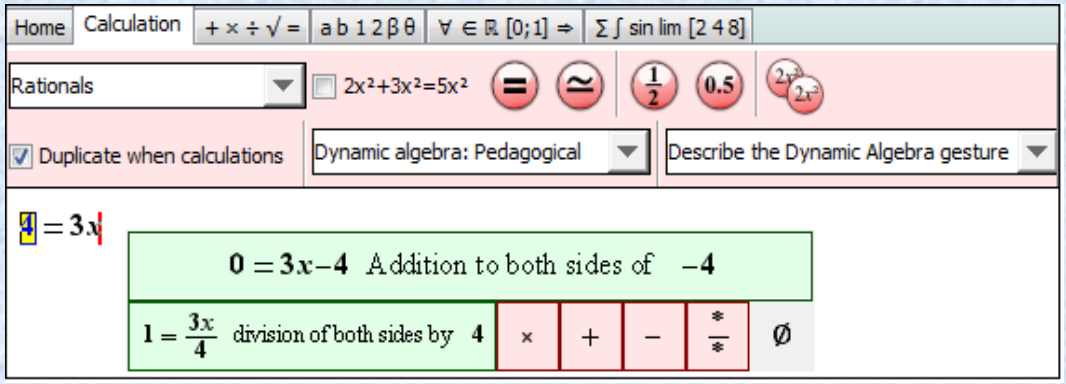
Many studies (Kieran, 2007; Wittmann et al., 2013) highlight that students use gestures.

Analyzing students doing together correct calculations, Wittmann et al. say:

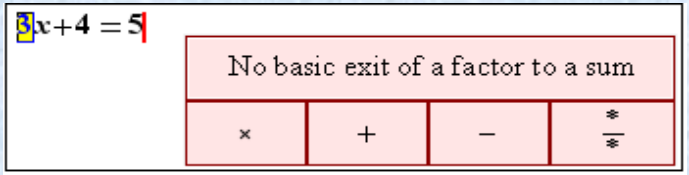
gestures and ambiguous speech of moving are the only algebra used at that moment

Implementation of the TMF in EpsilonWriter: Pedagogical Dynamic Algebra

A correct basic movement



An incorrect basic movement



Second Level of Dynamic Algebra:
More complex calculations with drag & drop

Equivalent drag & drop:

- 3 exits from $3x + 4$, providing $3\left(x + \frac{4}{3} \right)$
- 2 enters the parentheses in $(x - 3)^2$ to get $x^2 - 6x + 9$
- $(x - 2)^2$ is dropped over $(x - 2)^3$

  in $\frac{x^4(x-2)^2}{x(x-2)^3}$ to get $\frac{x^4}{x(x-2)}$

Non-equivalent drag & drop:

- $\begin{cases} x = 2a + 1 \\ y = a - 2 \end{cases}$ is dropped over $\frac{x-y}{x+2y}$ in order to apply a substitution, getting $\frac{2a+1-(a-2)}{2a+1+2(a-2)}$

Second Level of Dynamic Algebra

- Adding like terms
- Multiplying like factors
- Expanding
- Factorizing
- Simplifying
- Solving equations and inequalities
- Solving simultaneous equations

Third level of Dynamic Algebra: Click on Operators or Selections

Ctrl+Click to get an action when an action is available

- Completing the square and $a^2 - b^2$
- Solving schema for quadratic and cubic equations
- Table of signs
- Limits
- Derivatives
- Table of variations
- Definition conditions

Dynamic Algebra in the MC2 project

MC2 is a European project on Math Creativity

- At the present time, resources have been built for limits at infinity and the notion of infinity
- New resources for other domains will be built
- They will be experimented

Dynamic Algebra at IREM

IREM is a French association of Math Teachers

- Two groups start building resources, one for grades 6-9, one for grades 10-12
- They will also experiment the resources

EpsilonWriter

EpsilonWriter is Java software for Windows, MacOS and Linux

- running as applet on the website
- running as application (download on the website)

EpsilonWriter is free for non-commercial use

There is also an application for Chat with formulas and LIVE documents. It is distributed with EpsilonWriter.

http://epsilonwriter.com